

General Certificate of Education
January 2006
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Wednesday 25 January 2006 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. (1 mark)

(ii) Show that $f(-2) = 0$. (1 mark)

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where a and b are integers. (3 marks)

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of d . (3 marks)

2 A curve is defined by the parametric equations

$$x = 3 - 4t \quad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of t . (4 marks)

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4 marks)

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0 \quad (3 \text{ marks})$$

3 It is given that $3 \cos \theta - 2 \sin \theta = R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.

(a) Find the value of R . (1 mark)

(b) Show that $\alpha \approx 33.7^\circ$. (2 marks)

(c) Hence write down the maximum value of $3 \cos \theta - 2 \sin \theta$ and find a **positive** value of θ at which this maximum value occurs. (3 marks)

4 On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £ V , of the sculpture is modelled by the formula $V = Ak^t$, where t is the time in years since 1 January 1900 and A and k are constants.

- (a) Write down the value of A . (1 mark)
- (b) Show that $k \approx 1.07664$. (3 marks)
- (c) Use this model to:
- (i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; (2 marks)
- (ii) find the year in which the value of the sculpture will first exceed £800 000. (3 marks)

5 (a) (i) Obtain the binomial expansion of $(1 - x)^{-1}$ up to and including the term in x^2 . (2 marks)

(ii) Hence, or otherwise, show that

$$\frac{1}{3 - 2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x . (3 marks)

(b) Obtain the binomial expansion of $\frac{1}{(1 - x)^2}$ up to and including the term in x^2 . (2 marks)

(c) Given that $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ can be written in the form $\frac{A}{3 - 2x} + \frac{B}{1 - x} + \frac{C}{(1 - x)^2}$,
find the values of A , B and C . (5 marks)

(d) Hence find the binomial expansion of $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ up to and including the term in x^2 . (3 marks)

Turn over for the next question

Turn over ►

6 (a) Express $\cos 2x$ in the form $a \cos^2 x + b$, where a and b are constants. (2 marks)

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where a is an integer. (5 marks)

7 The quadrilateral $ABCD$ has vertices $A(2, 1, 3)$, $B(6, 5, 3)$, $C(6, 1, -1)$ and $D(2, -3, -1)$.

The line l_1 has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector \overrightarrow{AB} . (2 marks)

(ii) Show that the line AB is parallel to l_1 . (1 mark)

(iii) Verify that D lies on l_1 . (2 marks)

(b) The line l_2 passes through $D(2, -3, -1)$ and $M(4, 1, 1)$.

(i) Find the vector equation of l_2 . (2 marks)

(ii) Find the angle between l_2 and AC . (3 marks)

8 (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x - 6)^{\frac{1}{2}}$$

to find t in terms of x , given that $x = 70$ when $t = 0$. (6 marks)

(b) Liquid fuel is stored in a tank. At time t minutes, the depth of fuel in the tank is x cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x - 6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. (1 mark)

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. (2 marks)

END OF QUESTIONS